Exercise 2

Find the linearization $L(x)$ of the function at a.

$$f(x) = \sin x, \quad a = \pi/6$$

Solution

Start by finding the corresponding y-value to $x = \pi/6$.

$$f(\pi/6) = \sin\frac{\pi}{6} = \frac{1}{2}$$

Then find the slope of the tangent line to the function at $x = \pi/6$ by computing $f'(x)$,

$$f'(x) = \frac{d}{dx}(\sin x)$$

$$= \cos x,$$

and plugging in $x = \pi/6$.

$$f'(\pi/6) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Now use the point-slope formula to obtain the equation of the line going through $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ with slope $\sqrt{3}/2$.

$$y - f(\pi/6) = f'(\pi/6)(x - \pi/6)$$

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2}x - \frac{\pi\sqrt{3}}{12}$$

$$y = \frac{\sqrt{3}}{2}x - \frac{\pi\sqrt{3} - 6}{12}$$

Therefore, the linearization of the function $f(x)$ at $a = \pi/6$ is

$$L(x) = \frac{\sqrt{3}}{2}x - \frac{\pi\sqrt{3} - 6}{12}.$$

Below is a plot of the function and the linearization at $a = \pi/6$ versus x.